DECAY CHARACTERISTICS OF AIRBORNE TIME-DOMAIN ELECTROMAGNETIC DATA: WHERE TO FIND THE CONSTANT (TAU)

Tau Geophysical Consultants

Magdel Combrinck
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Outline

What is τ ?

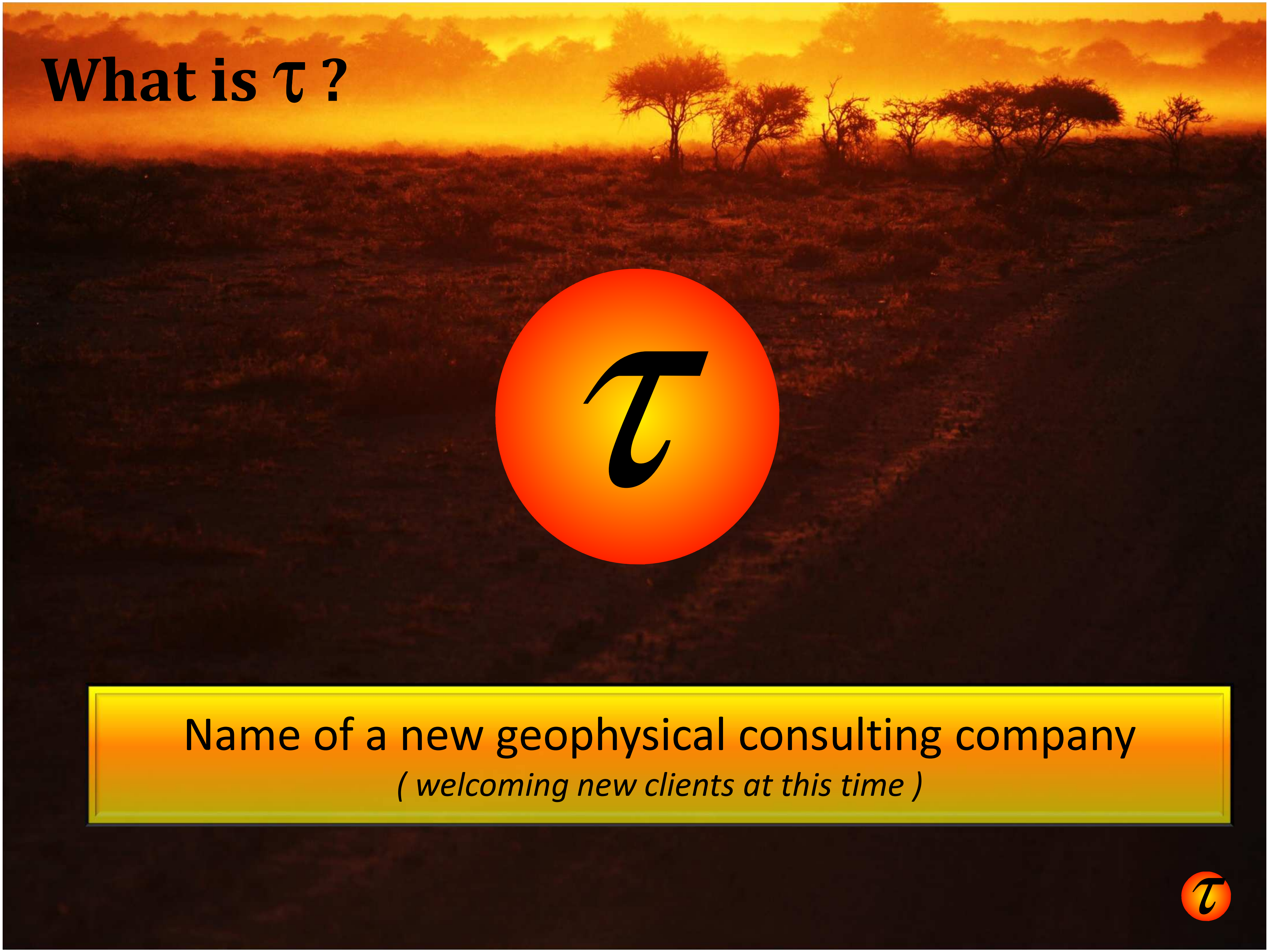
Why is it useful?

How is it calculated?

How do you explain different values for a “constant”?

… and which is the most accurate?


What is τ ?
Tswana for Lion

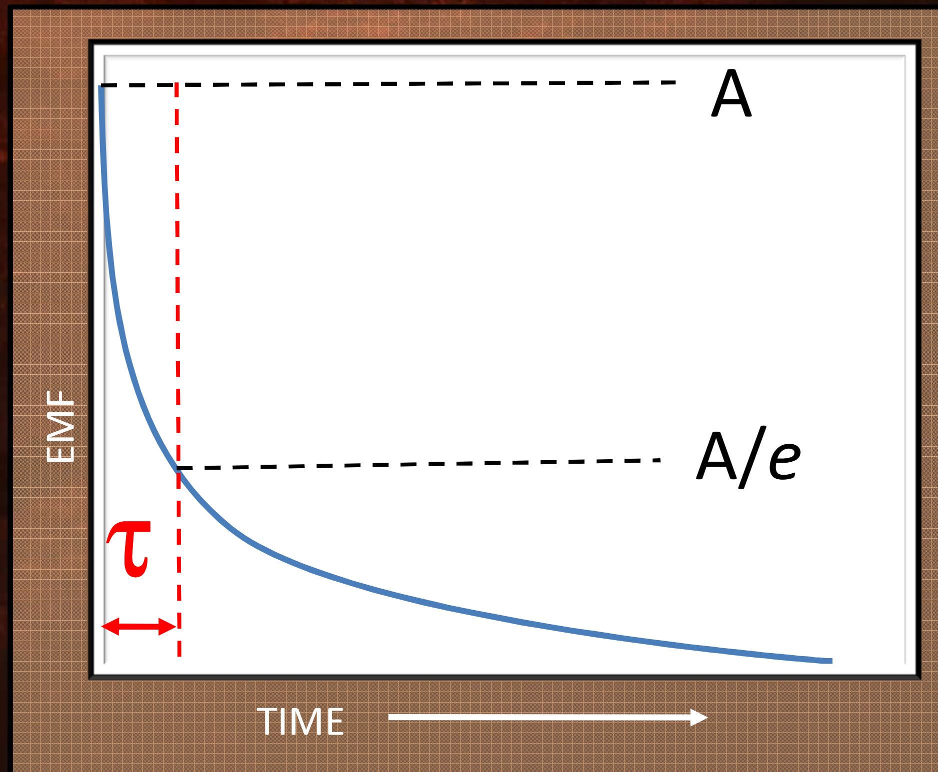

What is τ ?
τ
Name of a new geophysical consulting company
(welcoming new clients at this time)

What is τ ?

Decay Constant:

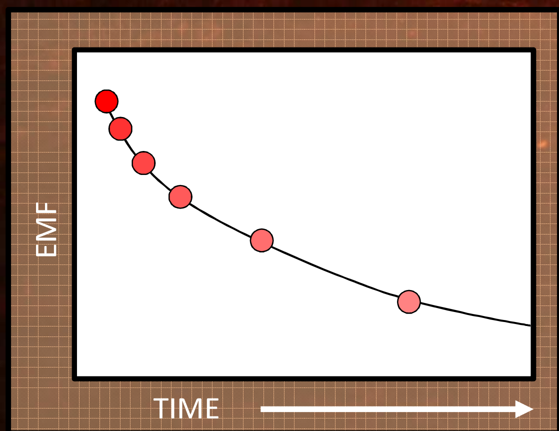
The time required for an exponential function to decay to a value of 1/*e* of the original value.

$$F(t)=Ae^{-t/\tau}$$

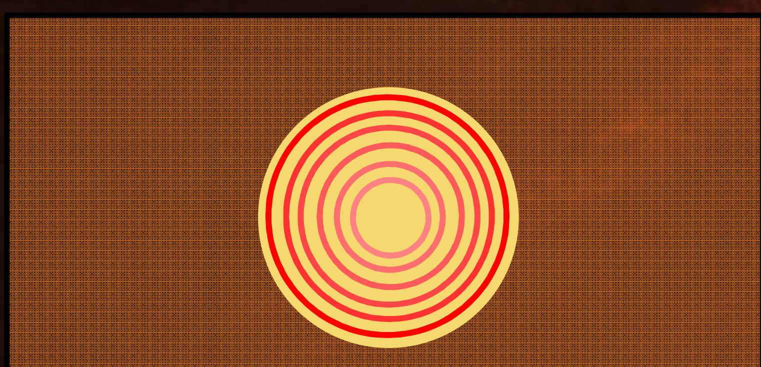


What is τ ?

In geophysics it is a parameter that links subsurface conductivity to transient electromagnetic field behaviour



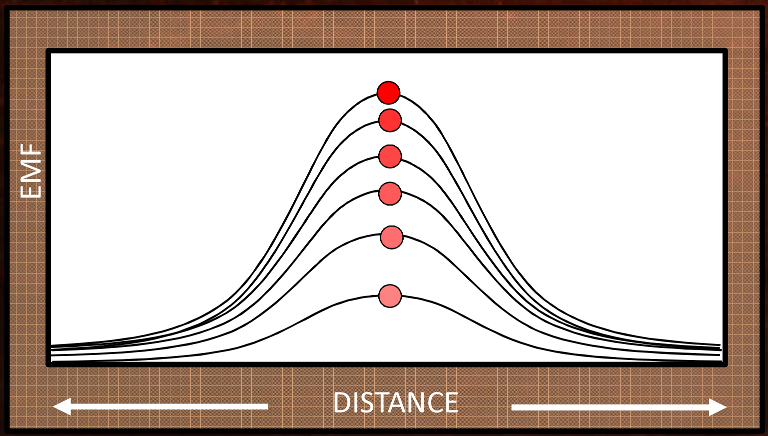
Measured EMF decay with time

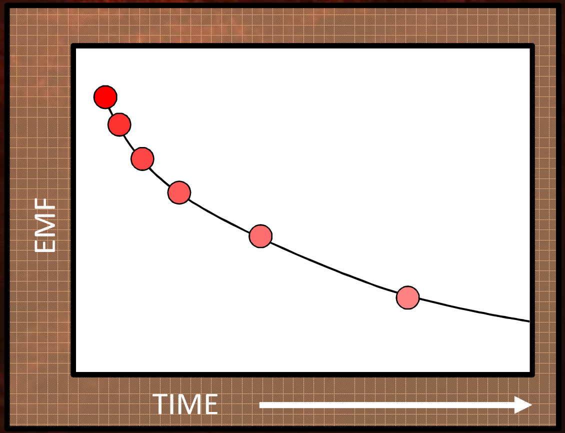


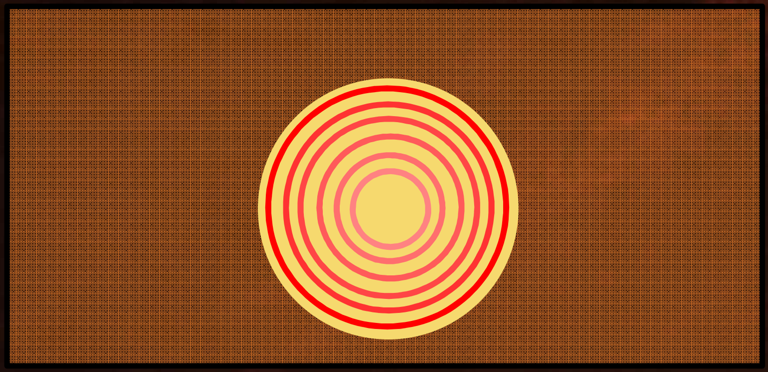
Subsurface Conductor

What is τ ?

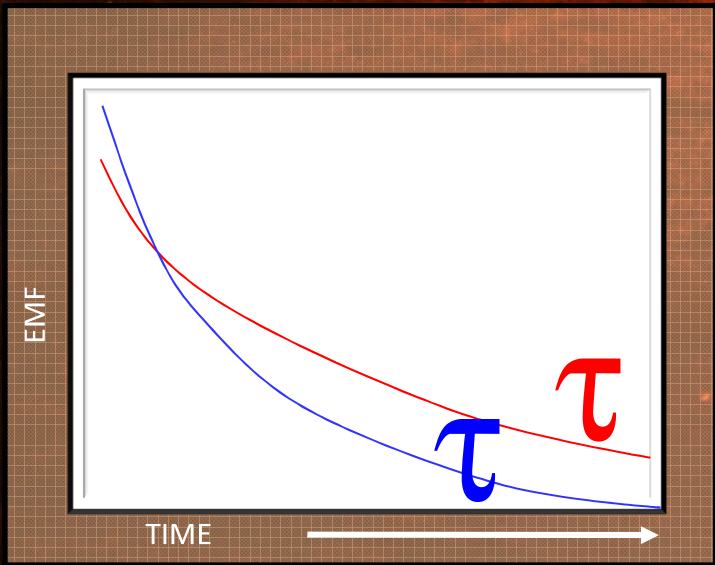
In geophysics it is a parameter that links subsurface conductivity to transient electromagnetic field behaviour







What is τ ?



$$B(t) = \sum_{i=0}^{n} A_i e^{-t/\tau_i}$$

$$EMF \propto \frac{dB(t)}{\partial(t)} = \sum_{i=0}^{n} \frac{A_i}{\tau_i} e^{-t/\tau_i}$$

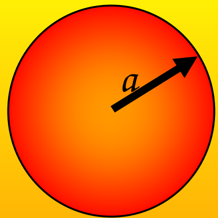
$\tau > \tau$

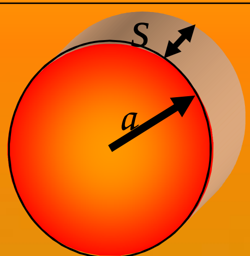
In late time:

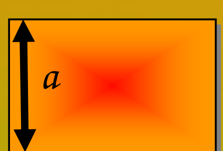
$$B(t) = Ae^{-t/\tau}$$

$$EMF \propto \frac{dB(t)}{\partial(t)} = \frac{A}{\tau} e^{-t/\tau}$$

What is τ ?

Model	Equation for τ	Equation for S
	$\tau = \frac{\sigma\mu a^2}{\pi^2}$	$S = \sigma a^2 = \frac{\tau\pi^2}{\mu}$

Model	Equation for τ	Equation for S
	$\tau = \frac{\mu a\sigma s}{5.51}$	$S = \sigma s = \frac{5.51\tau}{\mu a}$

Model	Equation for τ	Equation for S
	$\tau \approx \frac{\mu a\sigma s}{10}$	$S = \sigma s = \frac{10\tau}{\mu a}$

τ is mathematically related to conductor conductivity and geometry (conductance), and describes how TEM fields decay in response to a specific conductor:

Late time relationships have been calculated for various geometries where:

S: Conductance [S]
σ: Conductivity [S/m]
μ: 4π x 10^{-7} [NA^{-2}]
a & s: Geometrical dimensions [m]

1 From Geonics Technical Notes 7 (McNeill)
2 Rock and Mineral Properties (Keller; in Electromagnetic Methods in Applied Geophysics Vol.1)
3 Interpretation of large-loop transmitter TEM surveys, McNeill 1982.

Why is it useful?

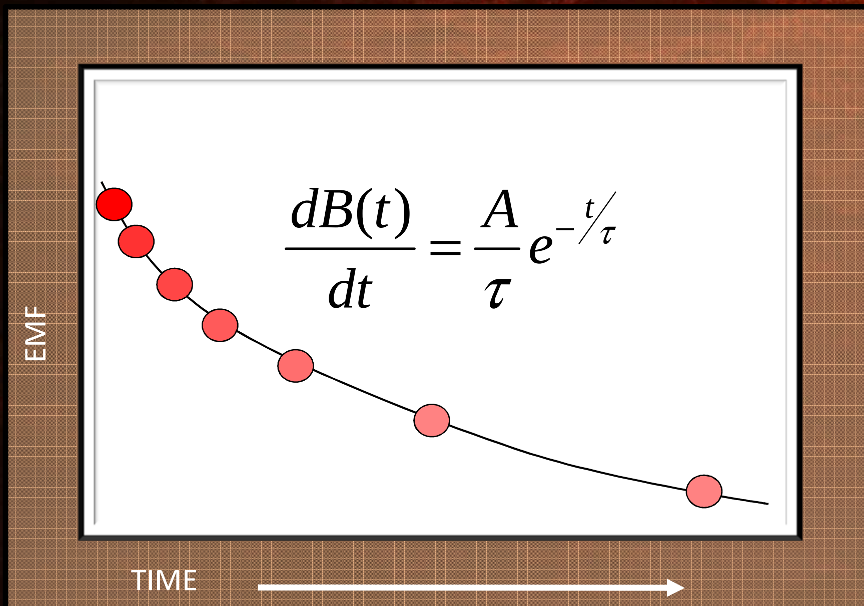
Analyzing TEM fields to determine decay characteristics provides subsurface conductivity information

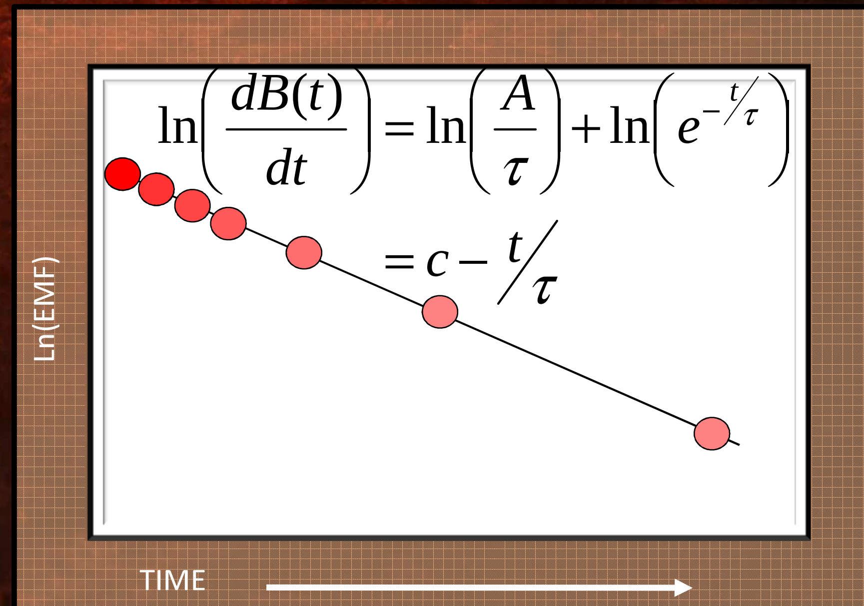
Decay constant maps can be produced vey quickly and are useful for a first pass evaluation of TEM data to indicate the approximate location and conductance of subsurface conductors.

It can also be used to delineate structure and lithological units.

How is it calculated?

1) Fit exponential function to dB(t)/dt or straight line to ln(dB(t)/dt); or use B(t) and ln(B(t))





2) Take ratio of late time B(t) to late time dB(t)/dt

$$\frac{B(t)}{\frac{dB(t)}{d(t)}} = \frac{Ae^{-t/\tau}}{\frac{A}{\tau} e^{-t/\tau}} = \tau$$

How is it calculated?

Plus some practical decisions

1) Which channels?

2) How many channels to evaluate at a time?

3) How to deal with noise?

4) When is the data fit acceptable?

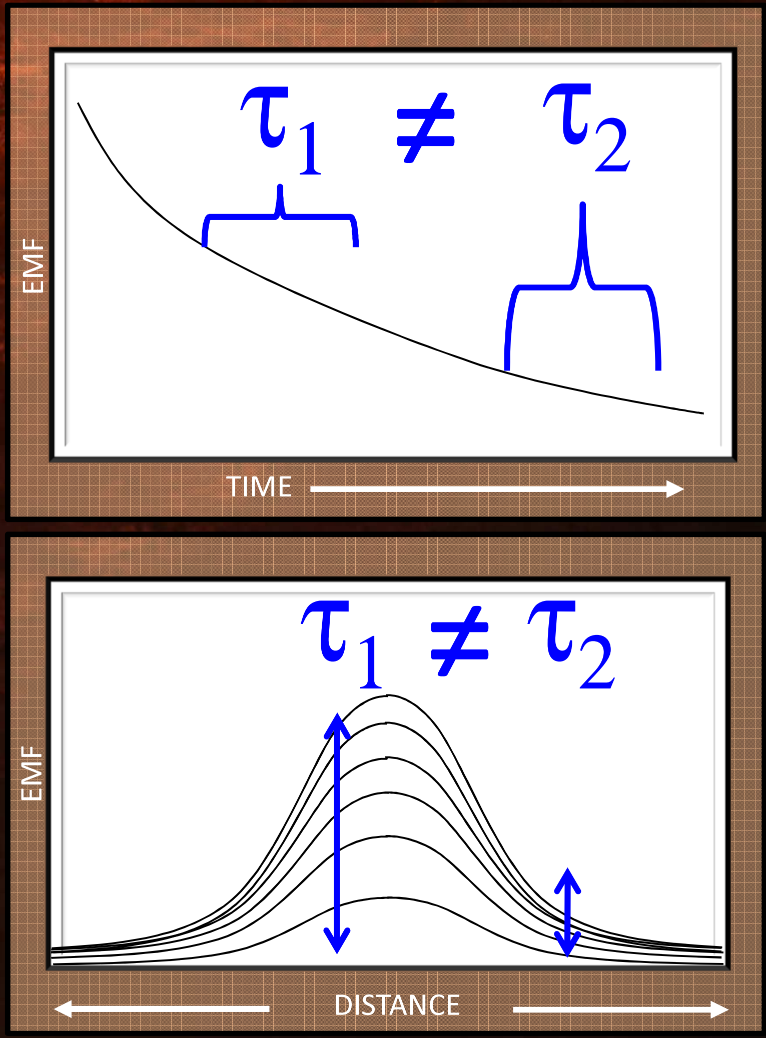
Different values for a constant?

As simple as these calculations are, some discrepancies are noticed when applying them to real data:

1) Often $\tau_{B(t)} \geq 2\,\tau_{dB(t)/\,dt}$

2) τ variation depending on which channels are selected for analysis

3) τ variation depending on which part of anomaly is selected for analysis



Different values for a constant?

Why?

1) Waveform?

2) Host rock with finite conductivity?

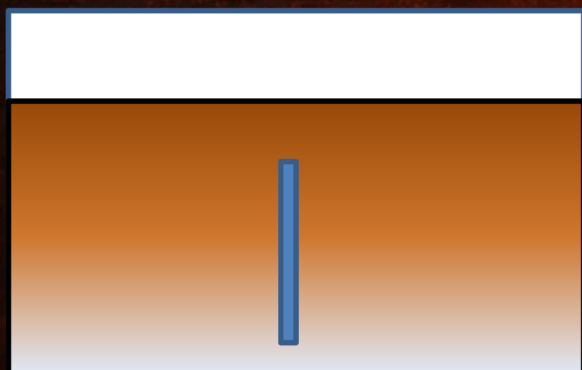
3) Do airborne systems ever measure in "late time"?

Which calculation is the most accurate ?

Methodology

Plate model in host rock with varying conductivity

(1,10,100,1000 , ∞ Ohm.m)



Depth to top: 50 m *Conductance: 100 S*

Strike Length: 200 m *Depth extent: 100 m*

Expected Tau: 1.26ms

Calculate response over 5 lines with Maxwell and Leroi Air

Varying …

dB(t)/dt	B(t)
Typical System Current Waveform	Theoretical Impulse Response

Total of 20 data sets

Calculate Tau using six different approaches on each model

Compare results

Methodology

Six approaches to calculate τ

“Exponential Fit Late Time”
3.5 – 8.0 ms

“Exponential Fit Mid Time”
1.0 – 2.4 ms

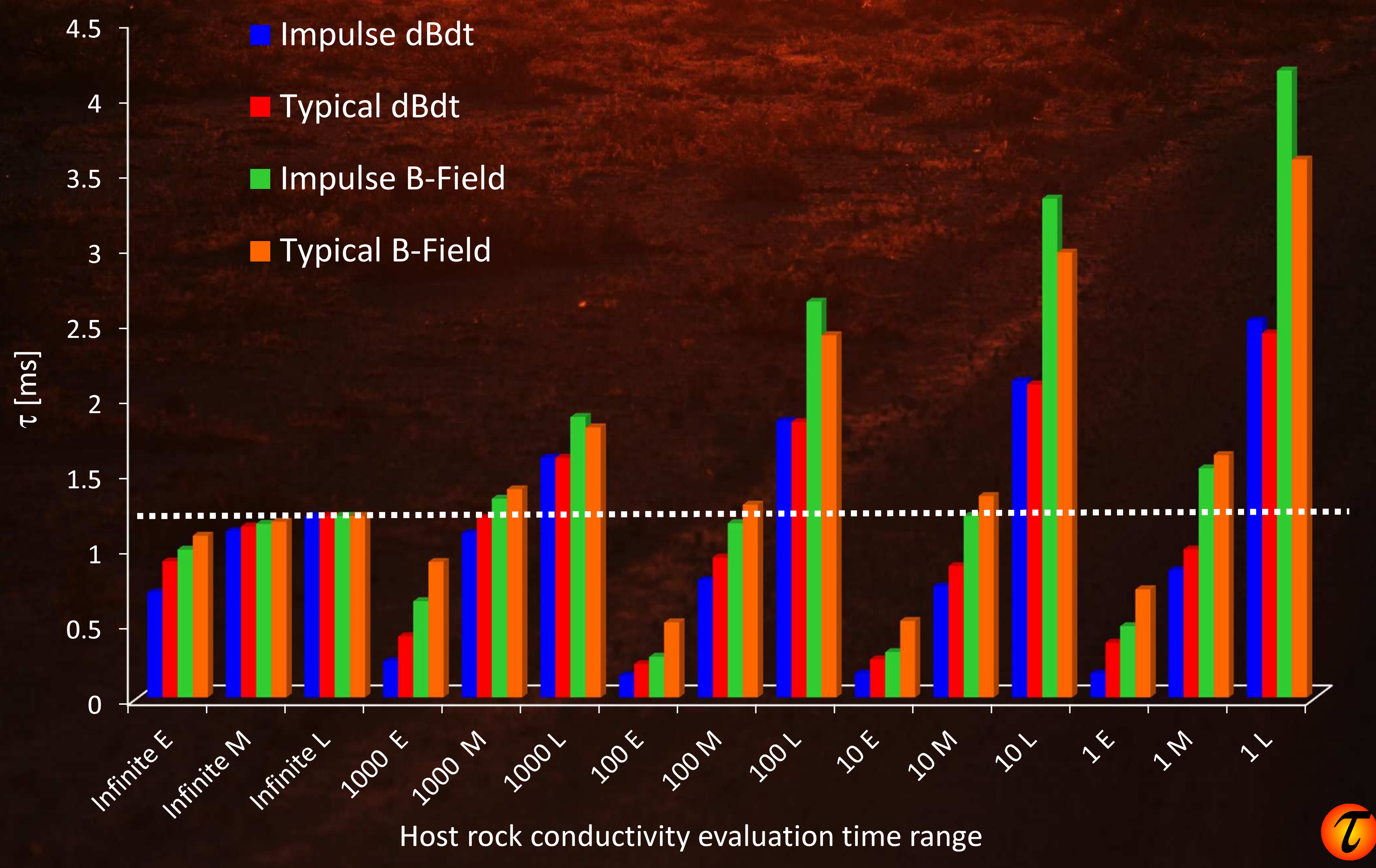
“Exponential Fit Early Time”
0.2 – 0.5 ms

“Ratio Tau”
Channel 32: [B(t)]/[dB(t)/ dt]
To compare to Ch 30-34 for other methods

“Latest Time Tau”
Most channels at latest time with r^2 fit better than 0.995

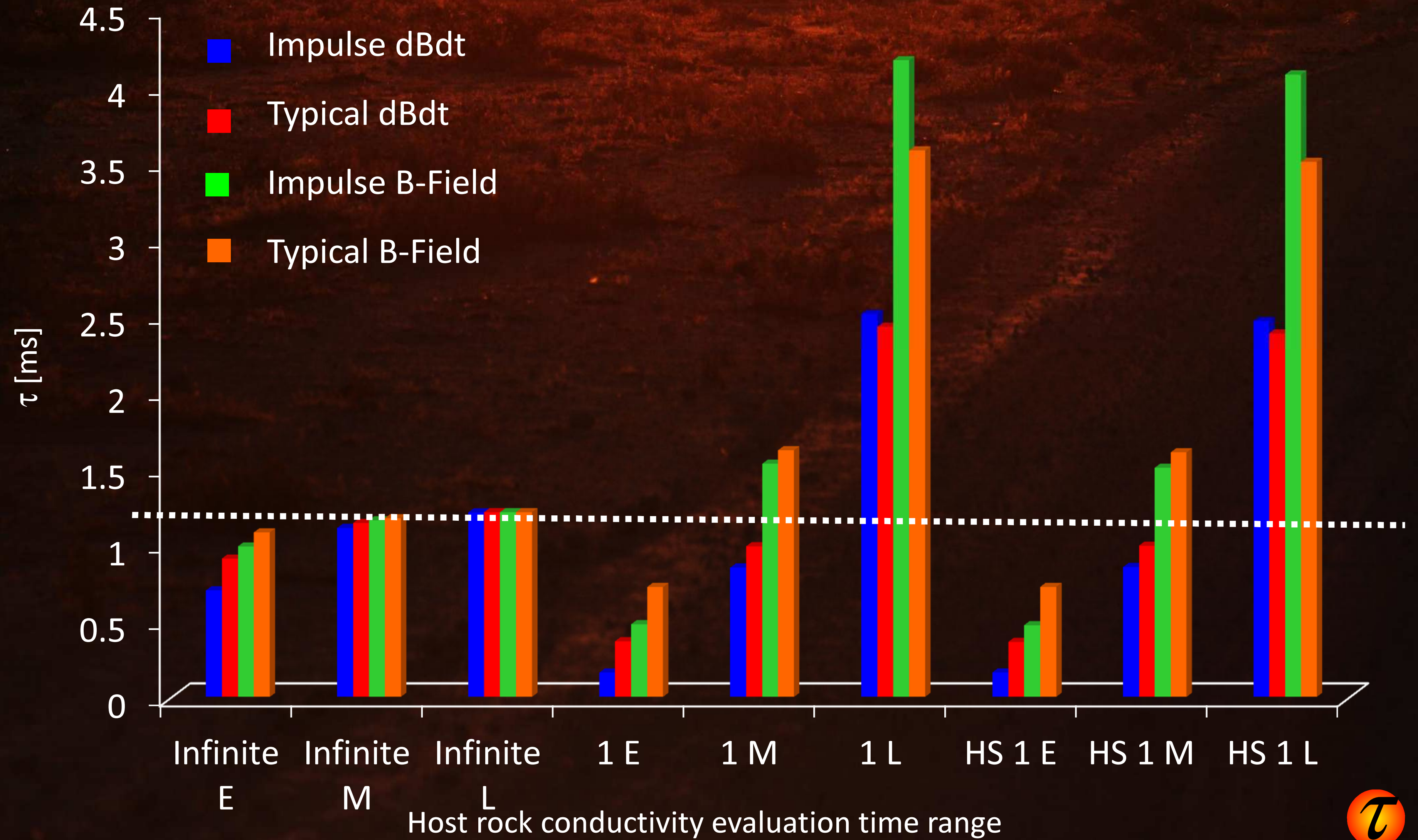
“Best Exponential Fit Tau”
Tau from any 5 subsequent channels showing best r^2 fit

Results



Impulse dBdt
Typical dBdt
Impulse B-Field
Typical B-Field
τ [ms]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Infinite E
Infinite M
Infinite L
1000 E
1000 M
1000 L
100 E
100 M
100 L
10 E
10 M
10 L
1 E
1 M
1 L
Host rock conductivity evaluation time range

Results



4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
τ [ms]
Impulse dBdt
Typical dBdt
Impulse B-Field
Typical B-Field
Infinite E
Infinite M
Infinite L
1 E
1 M
1 L
HS 1 E
HS 1 M
HS 1 L
Host rock conductivity evaluation time range

Methodology

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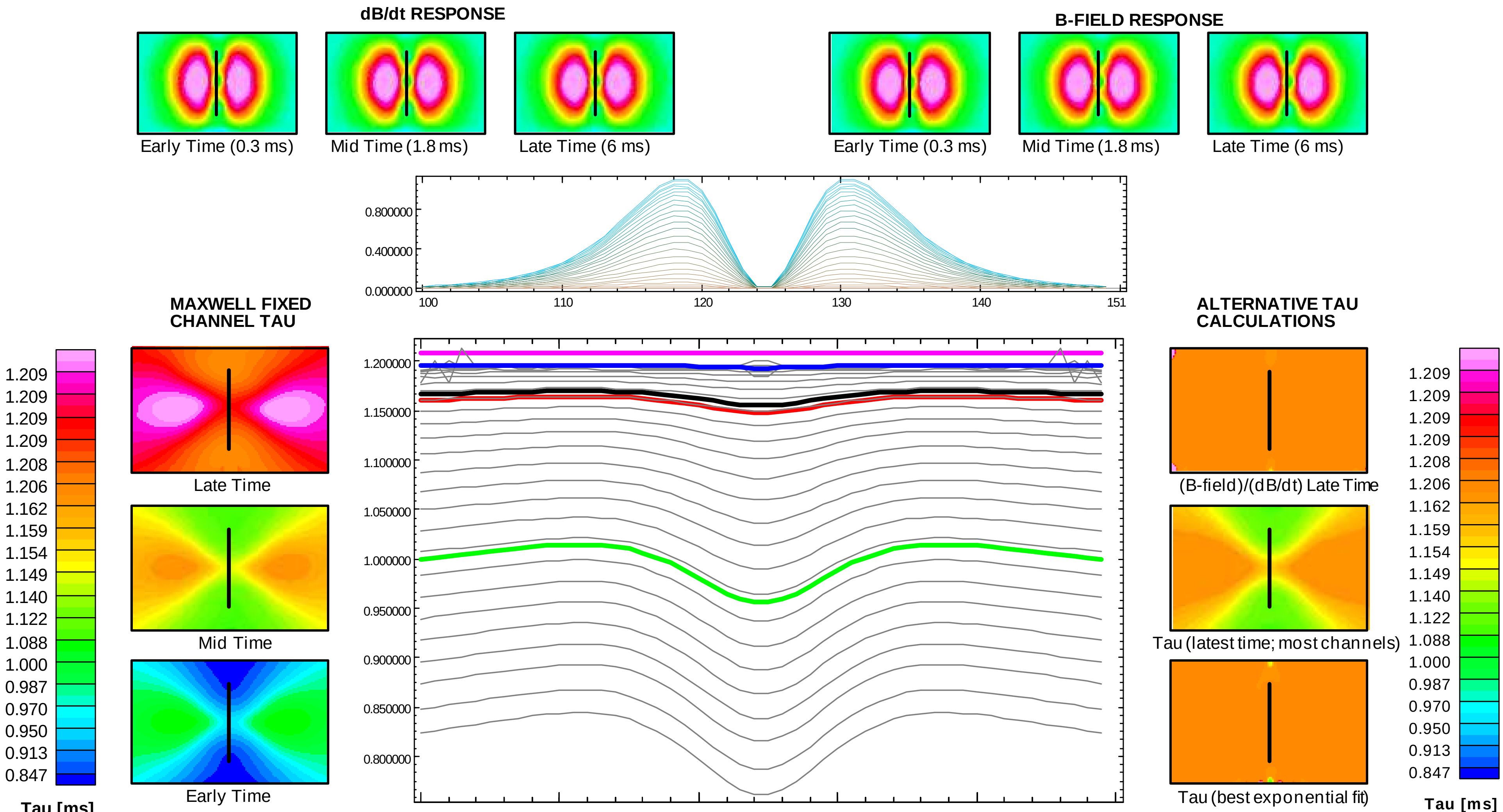
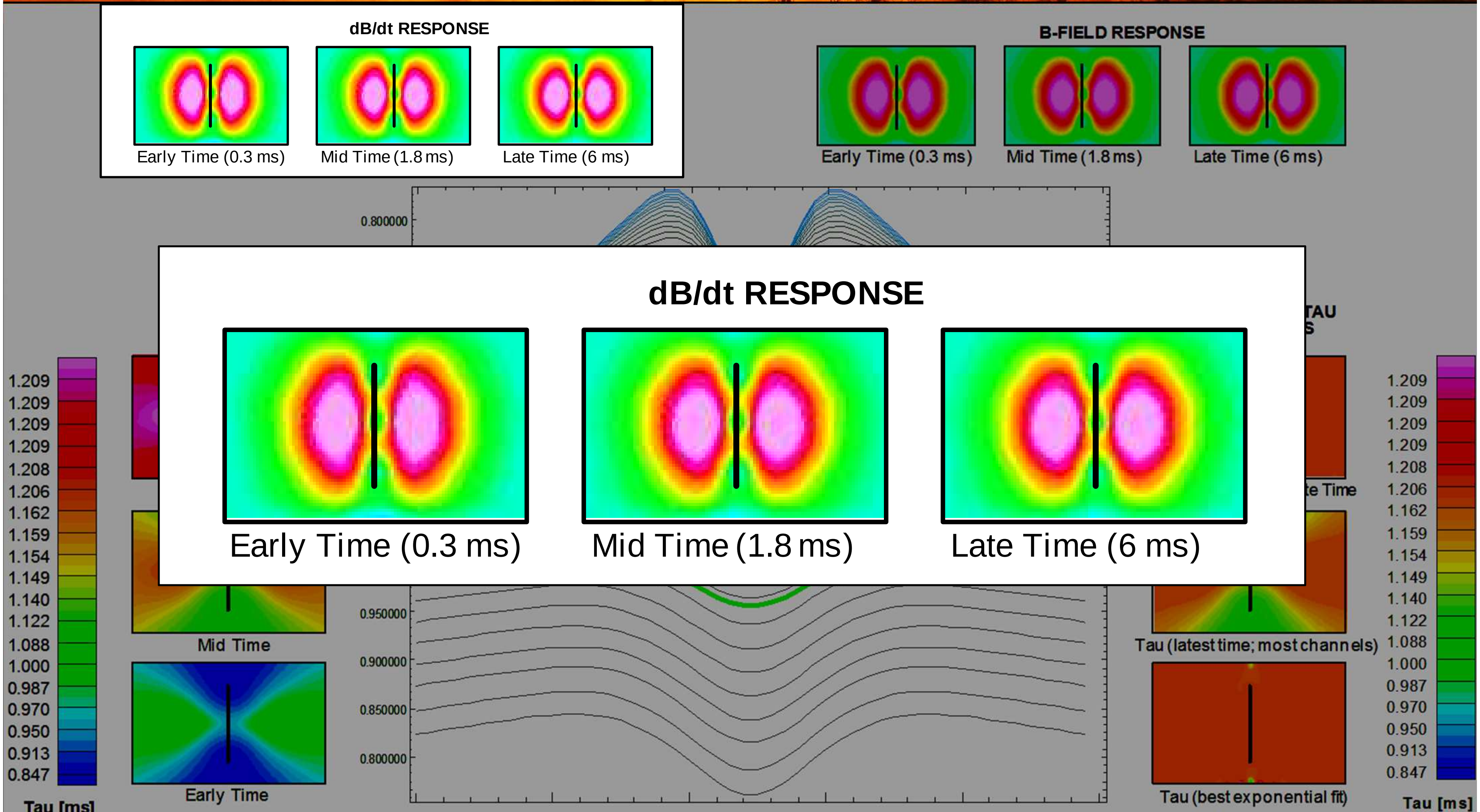
Results



dB/dt RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
B-FIELD RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
0.800000
0.400000
0.000000
100
110
120
130
140
151
MAXWELL FIXED CHANNEL TAU
1.209
1.209
1.209
1.209
1.208
1.206
1.162
1.159
1.154
1.149
1.140
1.122
1.088
1.000
0.987
0.970
0.950
0.913
0.847
Tau [ms]
Late Time
Mid Time
Early Time
1.200000
1.150000
1.100000
1.050000
1.000000
0.950000
0.900000
0.850000
0.800000
ALTERNATIVE TAU CALCULATIONS
(B-field)/(dB/dt) Late Time
Tau (latest time; most channels)
Tau (best exponential fit)
1.209
1.209
1.209
1.209
1.208
1.206
1.162
1.159
1.154
1.149
1.140
1.122
1.088
1.000
0.987
0.970
0.950
0.913
0.847
Tau [ms]

Results


dB/dt RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
B-FIELD RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
0.800000
dB/dt RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
1.209
1.209
1.209
1.209
1.208
1.206
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1.149
1.140
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1.088
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0.970
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0.913
0.847
Tau [ms]
Mid Time
Early Time
0.950000
0.900000
0.850000
0.800000
Tau (latest time; most channels)
Tau (best exponential fit)
1.209
1.209
1.209
1.209
1.208
1.206
1.162
1.159
1.154
1.149
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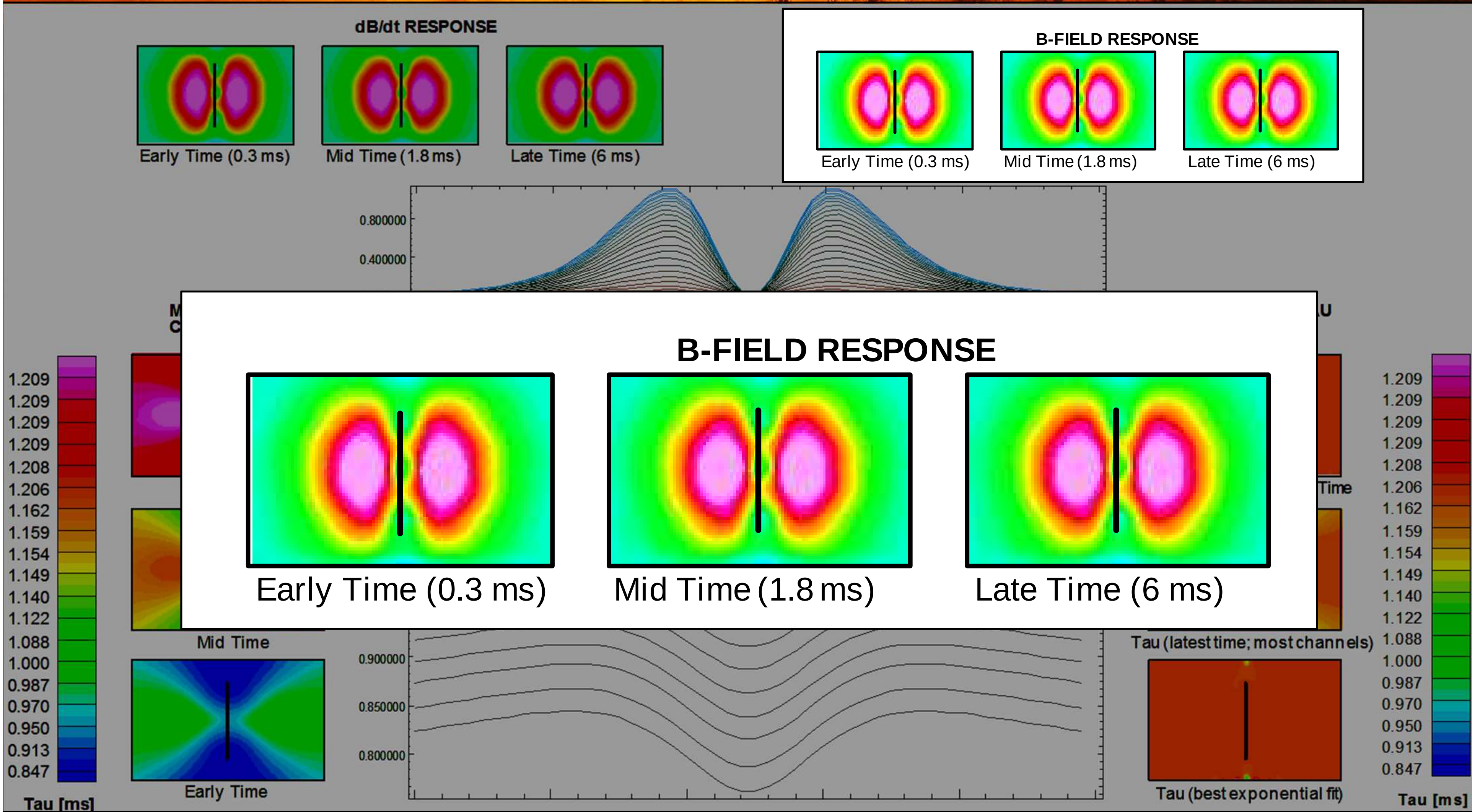
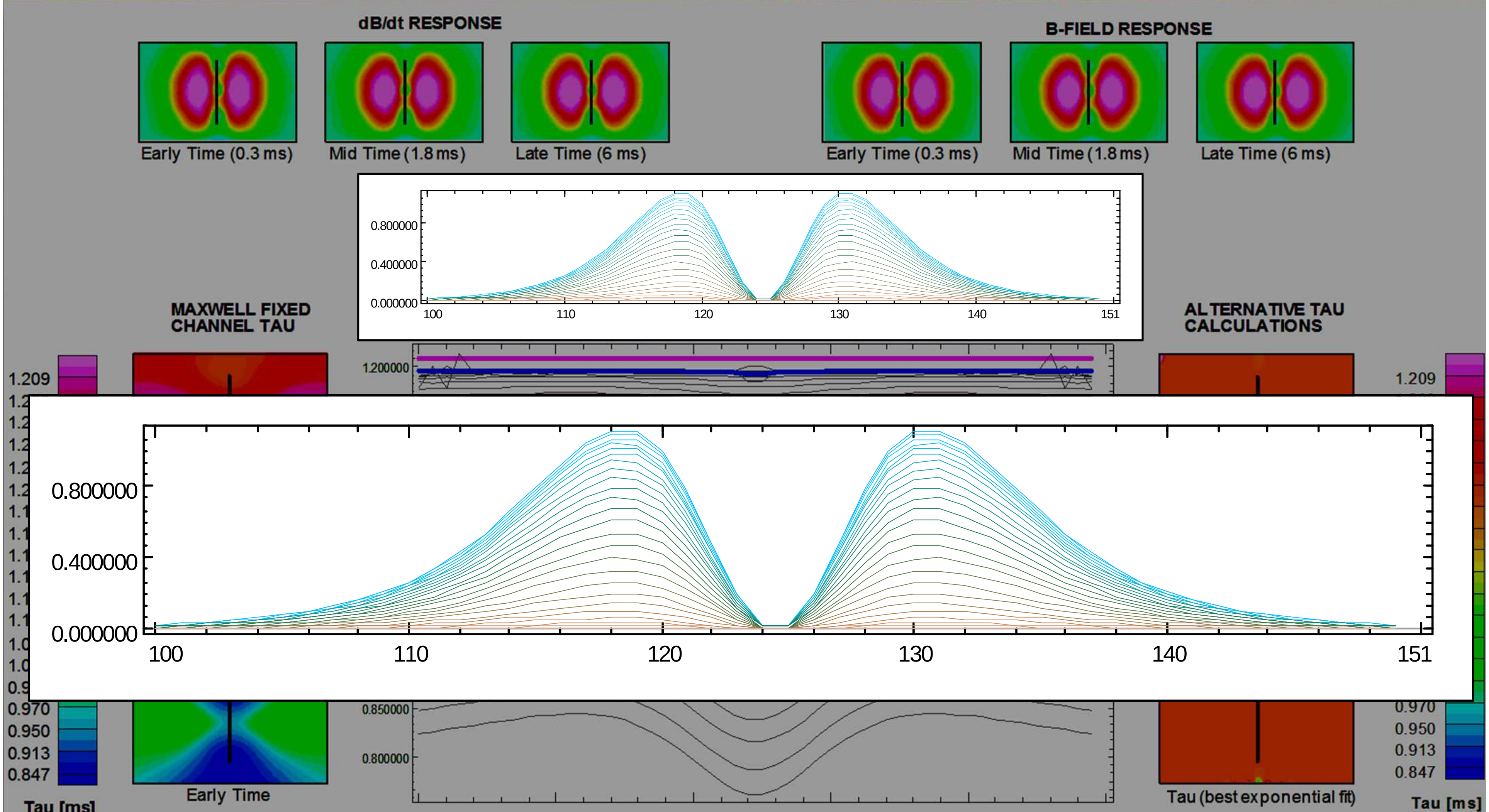
0.913
0.847
Tau [ms]

Results


dB/dt RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
B-FIELD RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
0.800000
0.400000
B-FIELD RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
Late Time (6 ms)
1.209
1.209
1.209
1.209
1.208
1.206
1.162
1.159
1.154
1.149
1.140
1.122
1.088
1.000
0.987
0.970
0.950
0.913
0.847
Tau [ms]
Mid Time
Early Time
0.900000
0.850000
0.800000
Time
Tau (latest time; most channels)
Tau (best exponential fit)
Tau [ms]

Results


dB/dt RESPONSE
B-FIELD RESPONSE
Early Time (0.3 ms)
Mid Time (1.8 ms)
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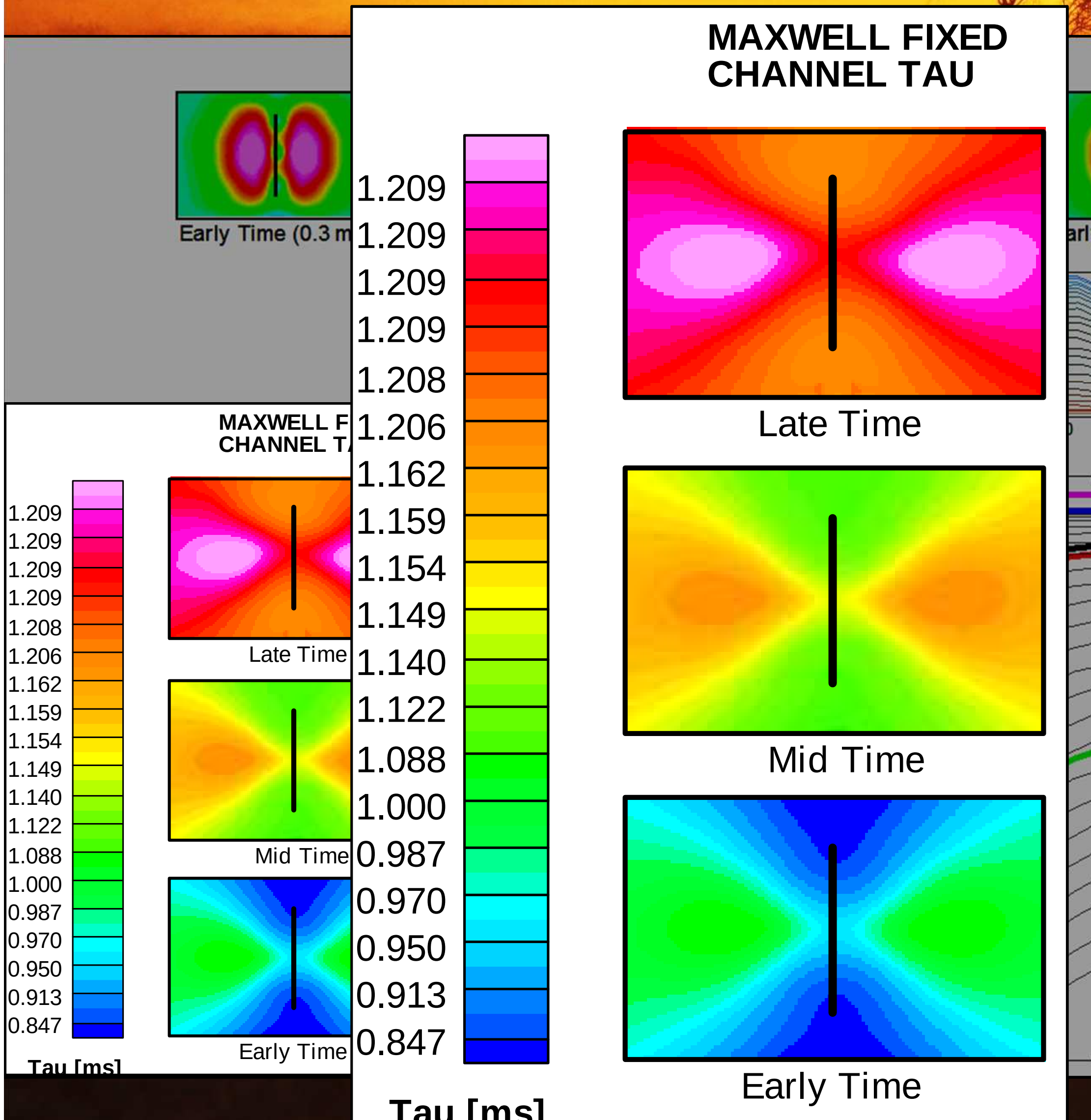
MAXWELL FIXED CHANNEL TAU
ALTERNATIVE TAU CALCULATIONS
0.800000
0.400000
0.000000
100
110
120
130
140
151
1.209
0.970
0.950
0.913
0.847
Tau [ms]
Early Time
Tau (best exponential fit)
1.200000
0.850000
0.800000

Results

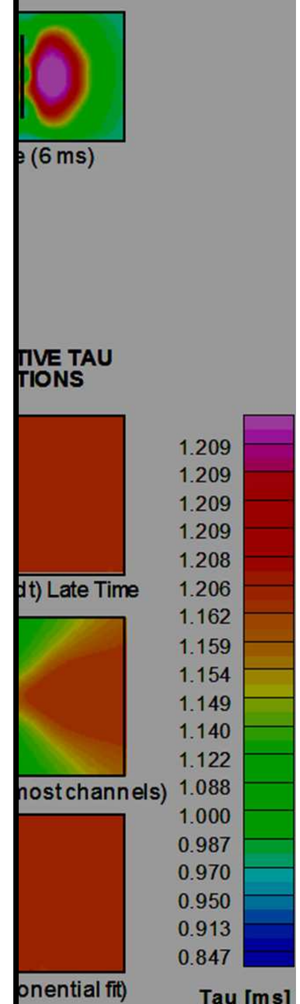


Forced fits, no test for r²

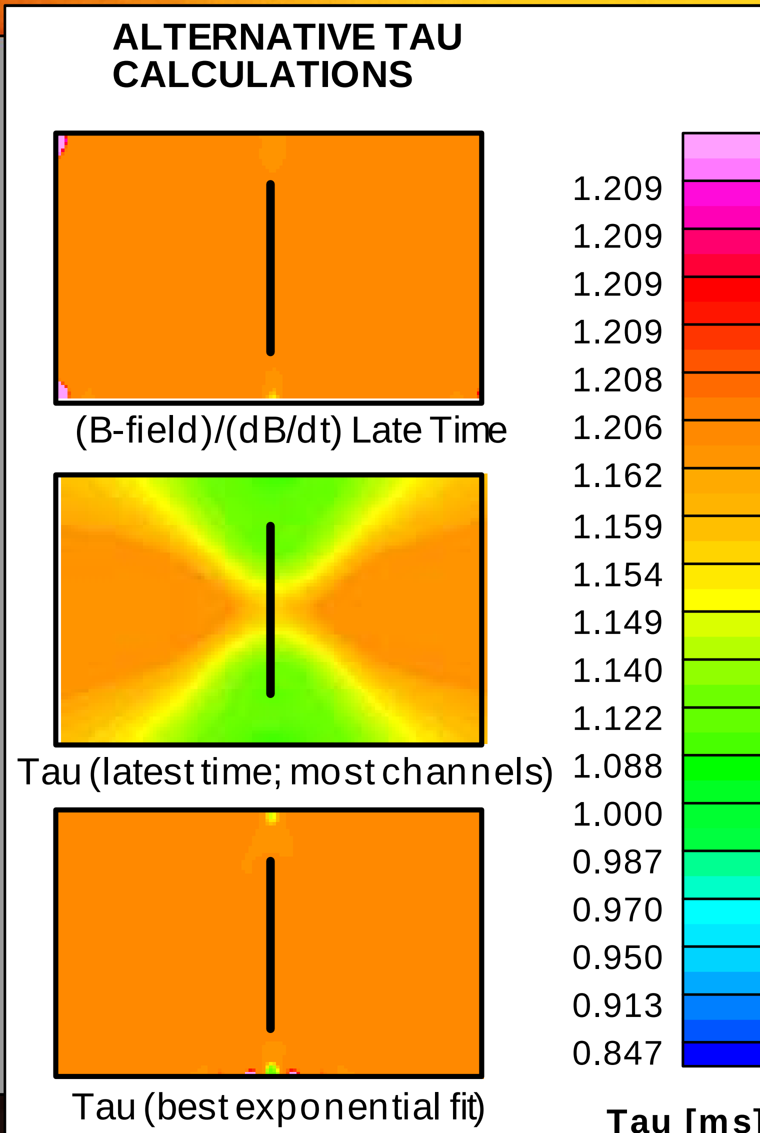
"Late Time"
3.5 – 8.0 ms
Ch 30 - 34

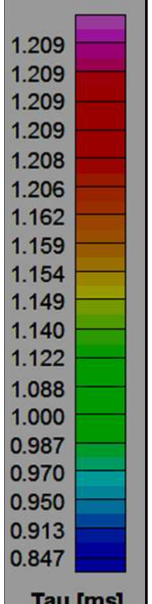
"Mid Time"
1.0 – 2.4 ms
Ch 23 - 27

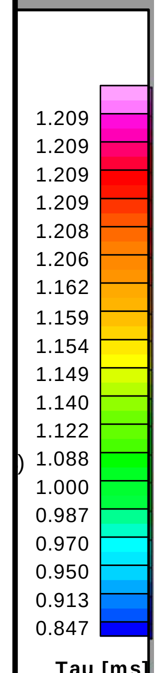
"Early Time"
0.2 – 0.5 ms
Ch 14 - 18



Results







“Ratio Tau”

Channel 32 B/(dB/dt)

To compare to Ch 30-34 for other methods

“Latest Time Tau”

Most channels (4 or more) at latest time with r^2 fit better than 0.995

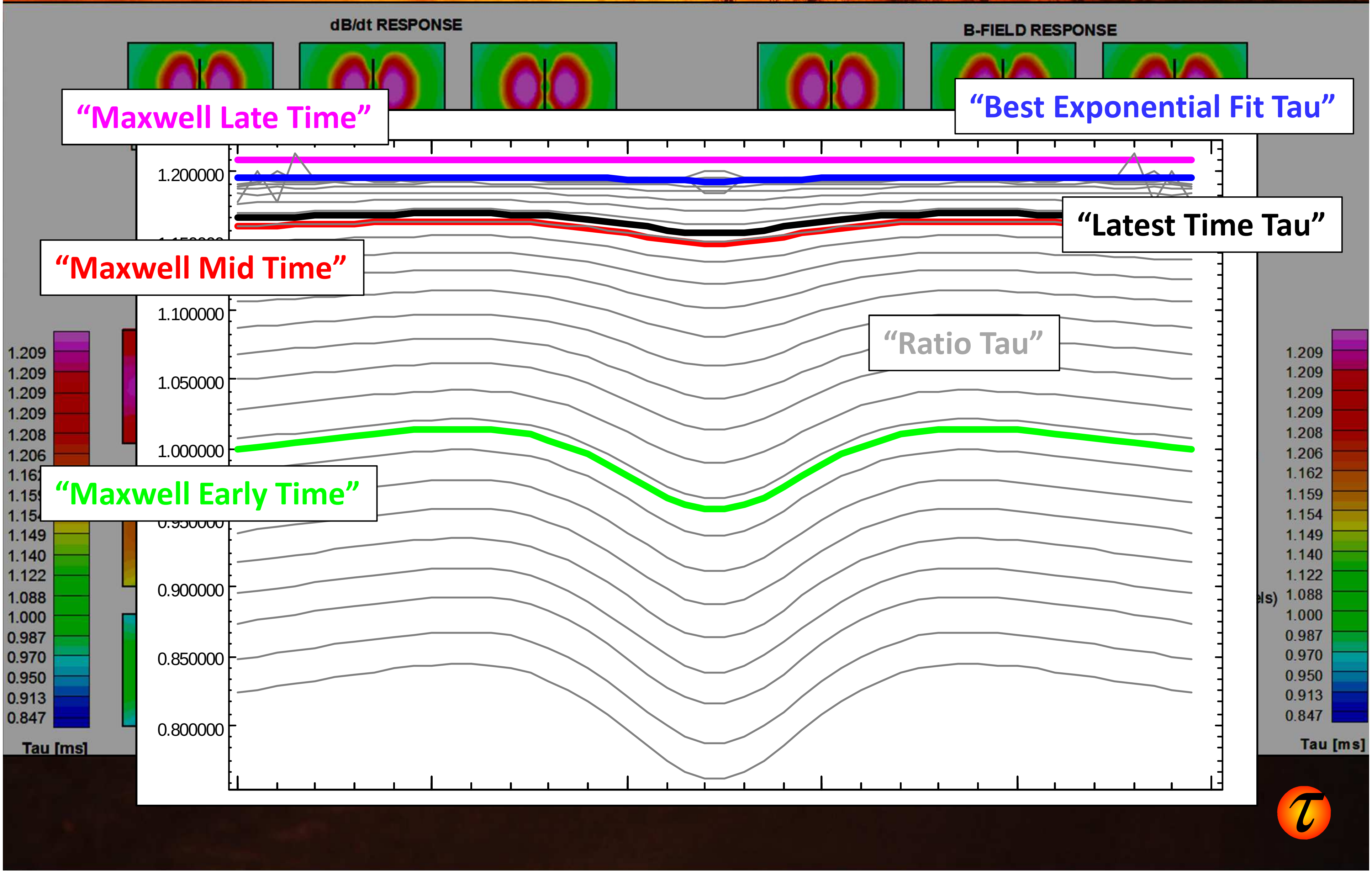
Channel 8-34: B-field

“Best Exponential Fit Tau”

Tau from any 5 consecutive channels showing best r^2 fit better than 0.995

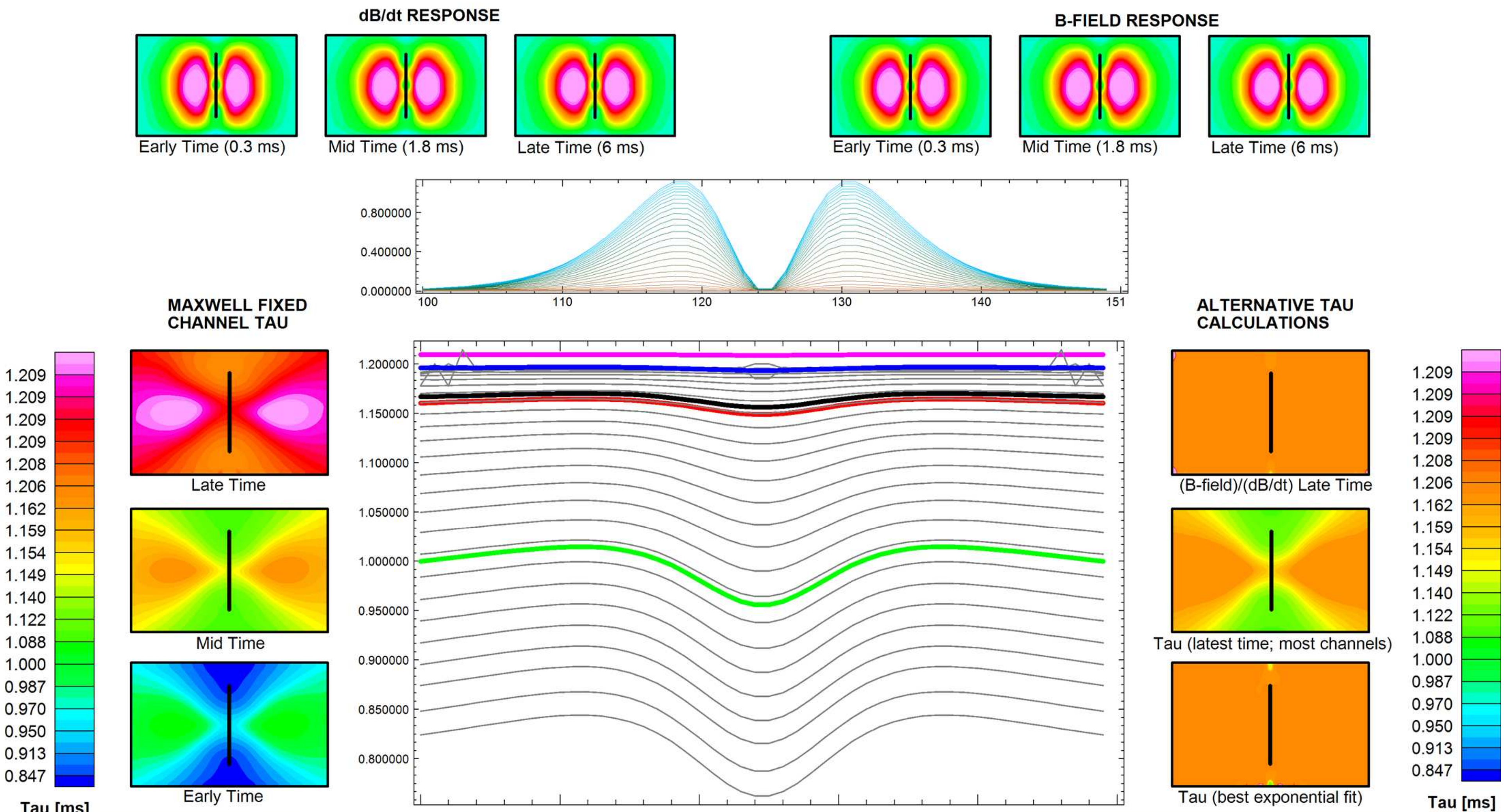
Channel 8-34: B-field

Results

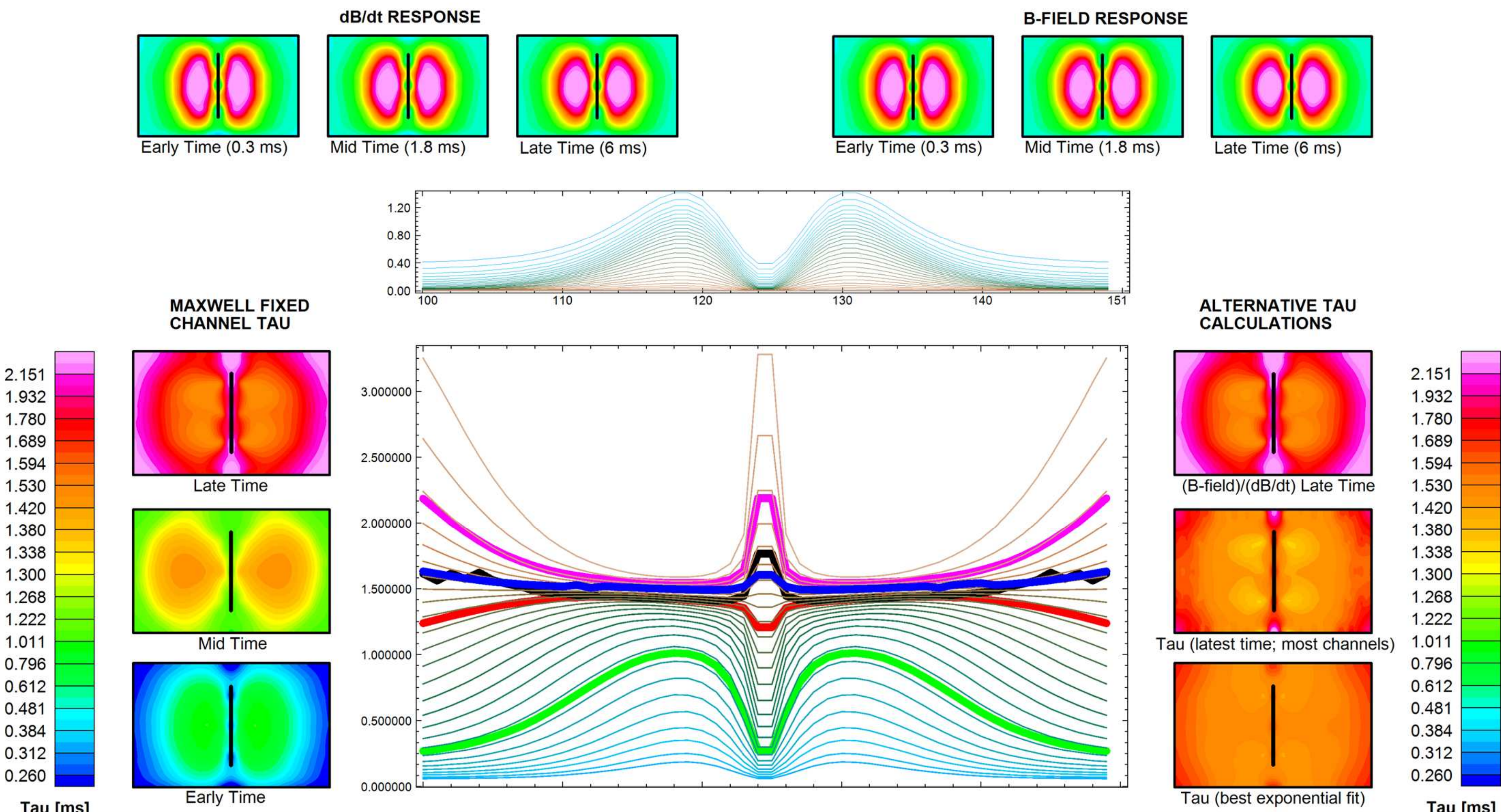


dB/dt RESPONSE
B-FIELD RESPONSE
"Maxwell Late Time"
"Best Exponential Fit Tau"
"Latest Time Tau"
"Maxwell Mid Time"
"Ratio Tau"
"Maxwell Early Time"
1.200000
1.100000
1.050000
1.000000
0.900000
0.850000
0.800000
1.209
1.209
1.209
1.209
1.208
1.206
1.162
1.159
1.154
1.149
1.140
1.122
1.088
1.000
0.987
0.970
0.950
0.913
0.847
Tau [ms]

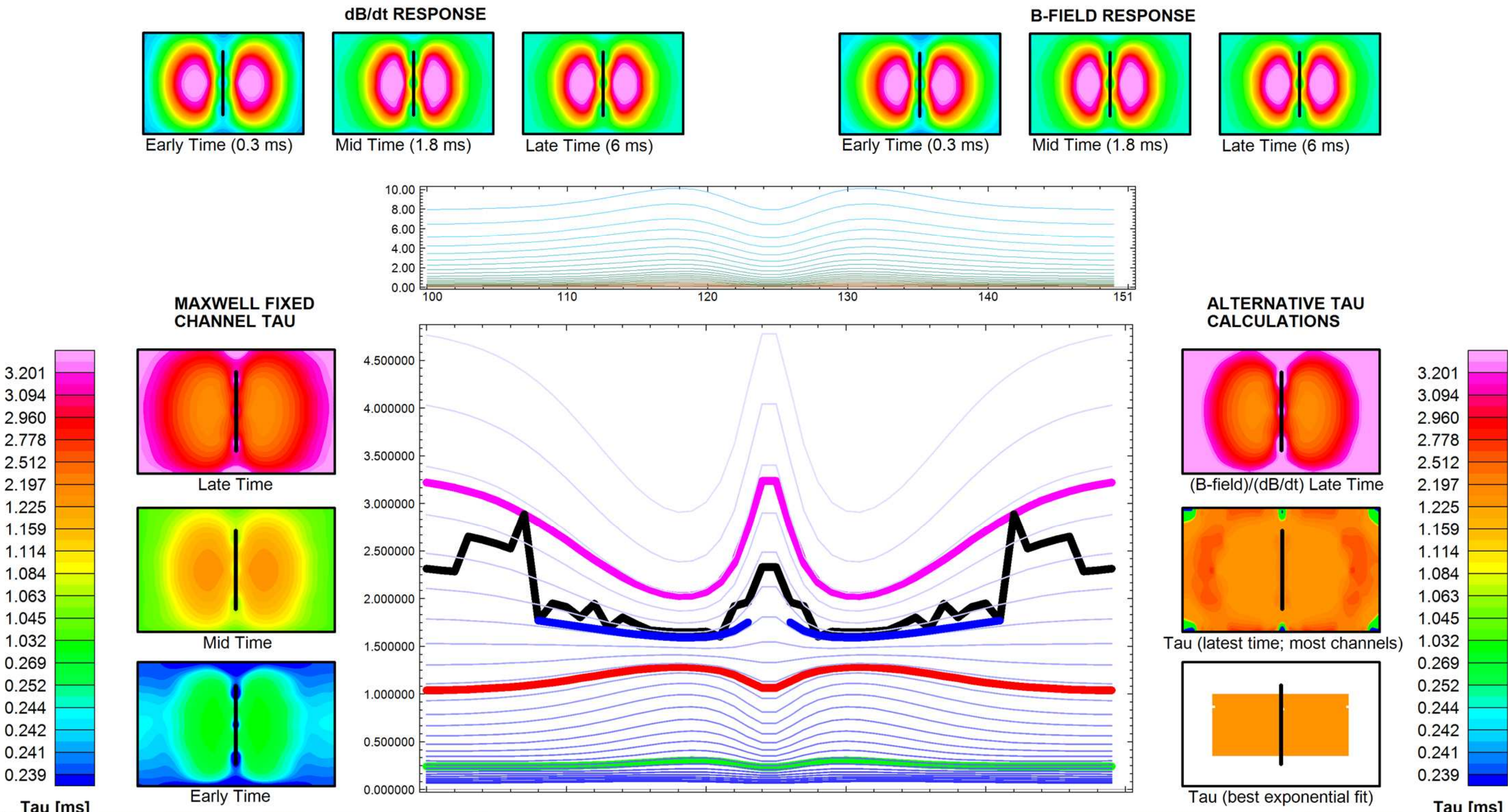
Infinite Resistivity



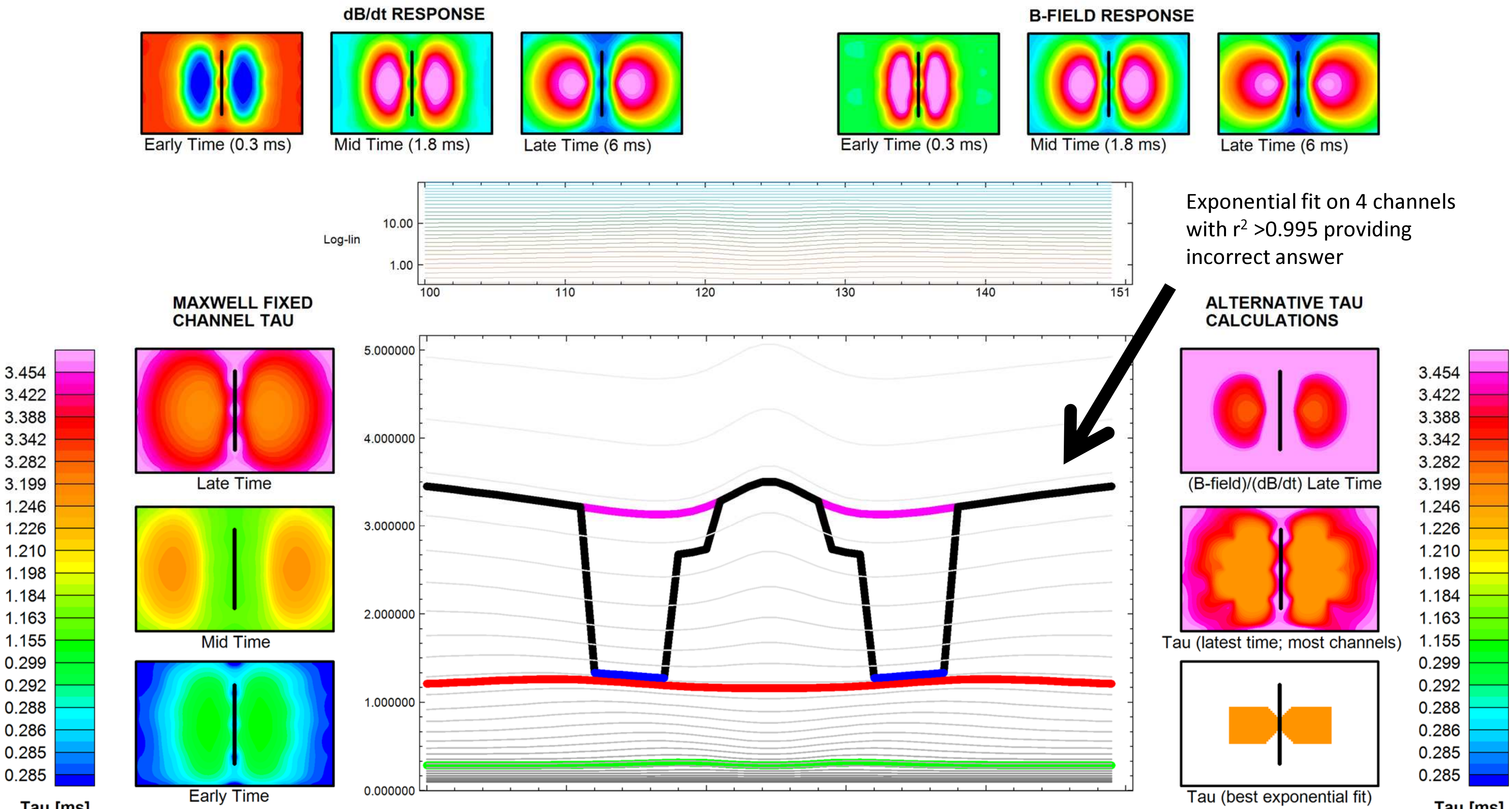
1000 Ohm.m host rock



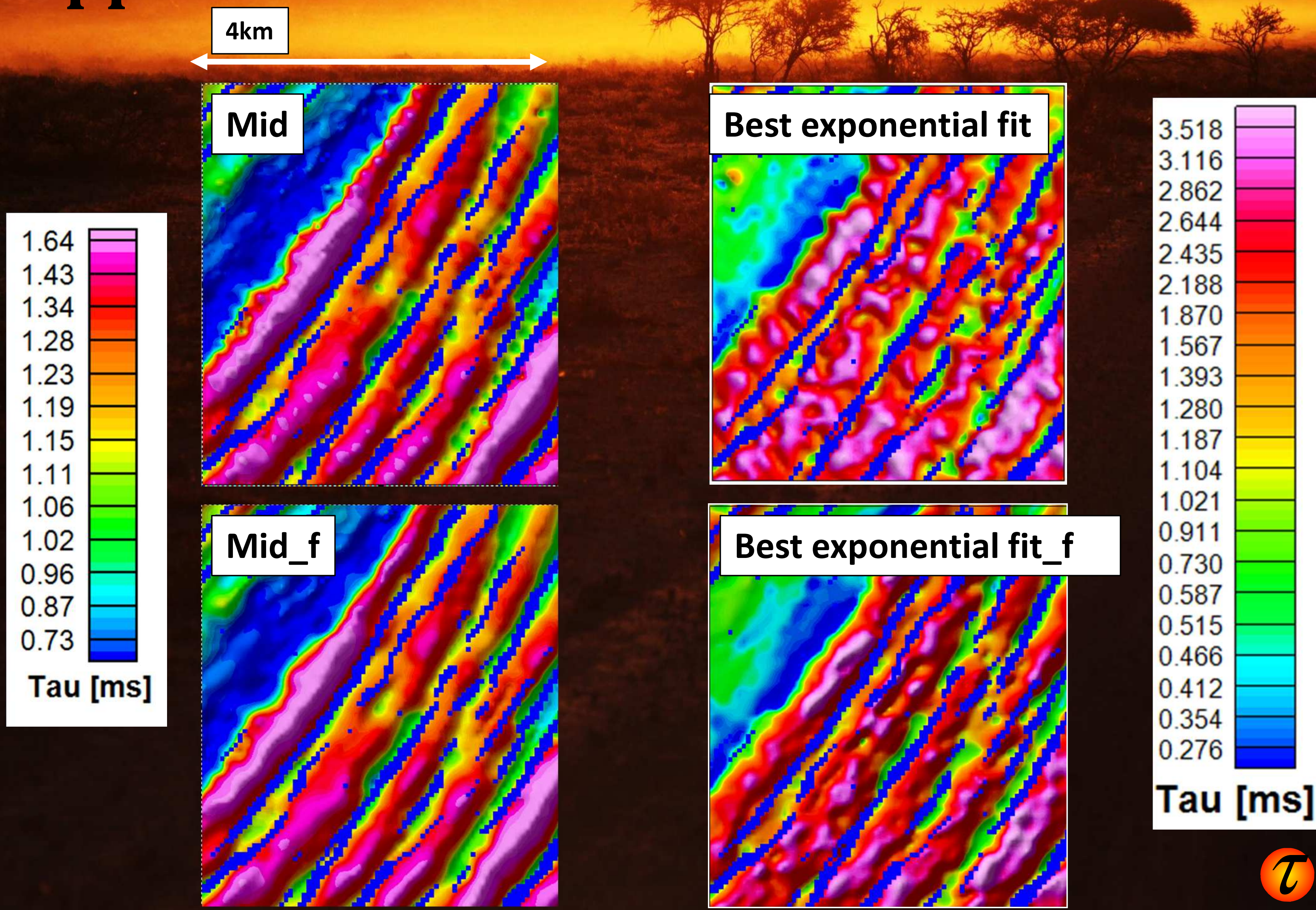
100 Ohm.m host rock



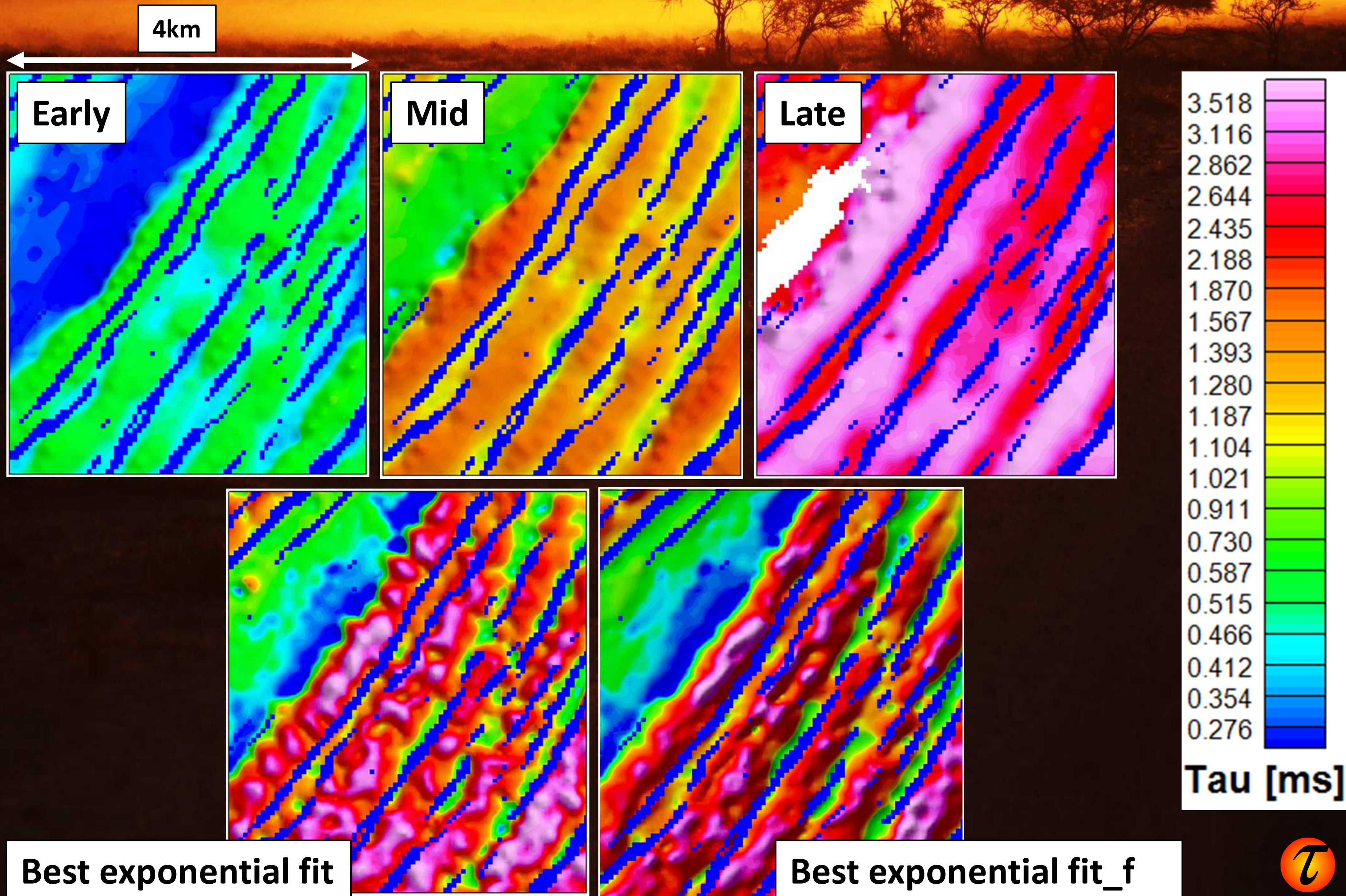
10 Ohm.m host rock



Application to field data



Application to field data



Conclusions

- There are different theoretical approaches to calculate τ

- In practice, when theoretical assumptions are not met i.t.o. system design and, more importantly, finite host rock conductivity the best approach is to find the position (in time and locality) where the decay is the closest to a single exponential decay

- This has to be constrained with great care as halfspace responses, in reality decaying proportional to $t^{-1.5}$ (B) and $t^{-2.5}$ (dB/dt) can easily be misinterpreted as exponential decays ($e^{-t/\tau}$), with regression coefficient fits (r^2) > 0.995 on 4 channels even though it is a power law decay. Using more than 4 channels are another way of constraining the result.

Thank you!

Thank you to Romex for use of field data.